High fidelity modeling of Euler buckling in fibrous networks captures reversible collapse of dendritic actin mesh

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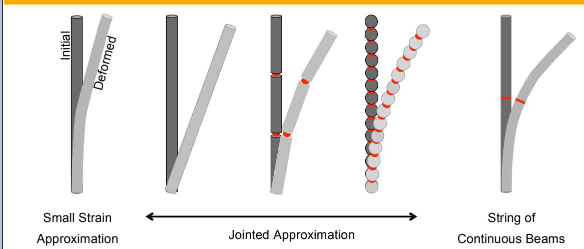
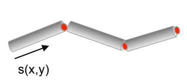
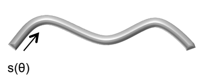
Abstract

From the cytoskeleton of cells to the transmission towers in the power grid, these structural frames consist of intricately arranged slender filaments. The deformation physics of these structures are largely governed by the bending of the slender members, which is a low resistance mode and can be quickly tipped to nonlinear levels even as the overall strains and forces remain small. While the physics of shear-free nonlinear bending has been described since the 18 century, simulations of arbitrary filamentous media have relied on linear and jointed approximations to the Euler bending equations. Our model comparisons show that these approximations stiffen the filament by an order and delay buckling. We present the **String-of-Continuous-Beams approach [1]** that captures the Euler buckling to fourth order continuity using just two nodes. The computational savings enables the translation to large network simulations of fibrous materials like tissues and cells. Complex crosslink force-transfers, such as that occurring between mother filament and daughter appendages, can be easily implemented. The approach was able to capture the nonlinear force and reversible collapse observed experimentally in compressed actin outcrops using seven generation of daughter filaments (147 microns of total actin length) and the computational power of a regular laptop.

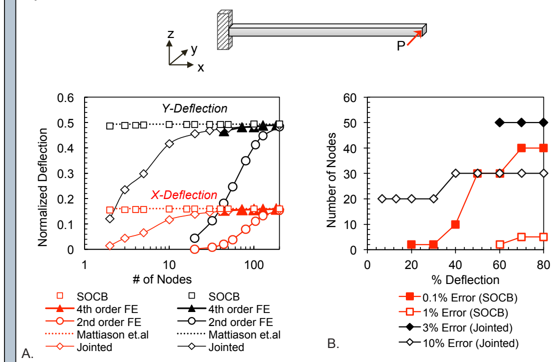
Philosophy of SOCB and Jointed Approach



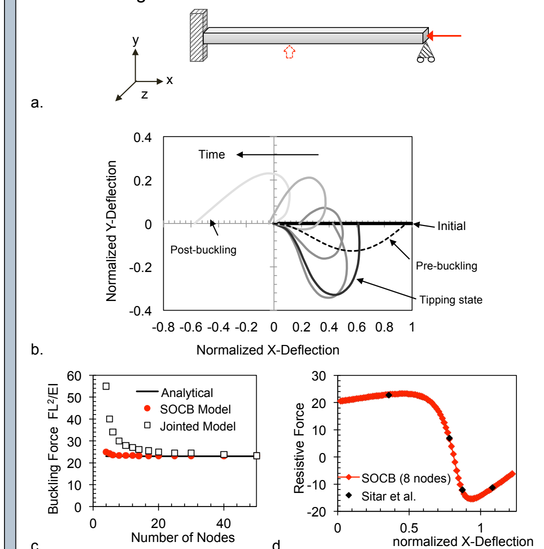
Conventional: String of Jointed Rods	Proposed: String of Continuous Beams
 s(x,y)	 s(θ)
Contour approximation: $\theta = {}^{\Delta y}/_{\Delta x}$	$x(s)=\int_0^s \cos\theta(s)\,ds;\ y(s)=\int_0^s \sin\theta(s)\,ds$
Mechanics-Geometry coupling: Moment and Force balance equations solve ${}^{\Delta y}/_{\Delta x}$, $\Delta\left[{}^{\Delta y}/_{\Delta x}\right]$	Moment and Force balance equations solve θ, ${}^{d\theta}/_{ds}$, and ${}^{d^2\theta}/_{ds^2}$
Nonlinearity in geometry comes from: Differences in θ at joints or disjointed bending of rigid section.	5^{th}-order continuous differential of θ within segment, and higher between segments.

Model Validation and Comparison to Jointed and FE Approaches:

1. Transverse bending of a beam: [A] Deflection vs. node number predicted for same loading; [B] Nodes # for reaching prescribed error at different deflection %.



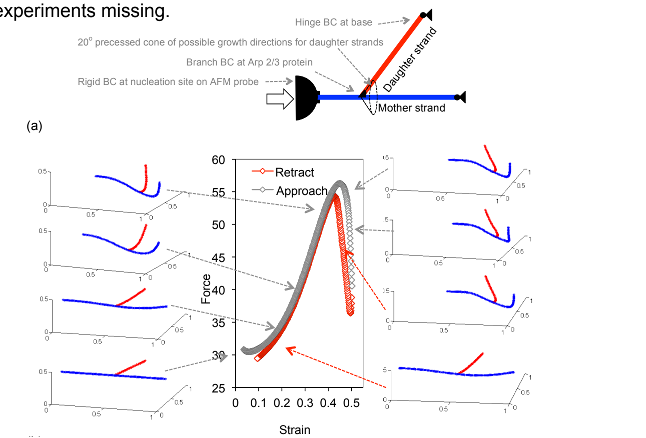
2. Axial buckling of a beam: [a] Loading ,[b] Buckling occurs after tipping state, [b] Tipping force vs. node # for SOCB and jointed approach, [c] Force vs. deflection during axial buckling in SOCB and detailed solution matches .



References: 1. Simhadhri and Chandran, Scientific Reports, 2019; 2. Chaudhari et al, Nature Communications, 2010.

Validation against experiment [2] on compression of dendritic actin outcrop

3. Compression of unit dendritic actin: [a] Schematic of mesh; [B] Addition of daughter branch changes axial resistance from linear increase and collapse (see Fig. 2d) to nonlinear increase and sharp collapse. Initial linear increase observed in experiments missing.



4. Compression of dendritic actin with 8 generations of branching: [a,b] Initial and 'collapsed' outcrop; [c] Outcrop force vs. strain (black diamonds) shows all experimental regimes (red diamonds) with an additional convex region; [d] Forces at different strains on all strands show only few load-bearing strands